Section 5.1 (Day 1) – Practice with Identities!

Recall: the first few identities we learned:

$$\sin x=\frac{1}{\csc x} \quad \cos x=\frac{1}{\sec x} \quad \tan x=\frac{1}{\cot x}$$

$$\csc x=\frac{1}{\sin x} \quad \sec x=\frac{1}{\cos x} \quad \cot x=\frac{1}{\tan x}$$

$$\tan x=\frac{\sin x}{\cos x} \quad \cot x=\frac{\cos x}{\sin x}$$

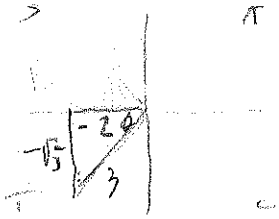
$$\sin^2 x+\cos^2 x=1 \quad 1+\tan^2 x=\sec^2 x \quad 1+\cot^2 x=\csc^2 x$$

First, we are going to use them in a way that should feel familiar – use one trig function to find all six!

Example: Let $\sec x=\frac{-3}{2}$ and $\tan x=\frac{\sqrt{5}}{2}$ to find the values of all six trig functions:

(in this one you are using the reciprocal identities)

$\sec x = \frac{1}{\cos x} = -\frac{3}{2} \Rightarrow \cos x = -\frac{2}{3}$



$\sin x = -\frac{\sqrt{5}}{3}$

$\cos x = -\frac{2}{3}$

$\tan x = \frac{\sqrt{5}}{2}$

$\csc x = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$

$\sec x = -\frac{3}{2}$

$\cot = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

Now we are going to use them to **simplify** expressions:

a) $\sin x+\cot x\cos x$

$= \sin x + \frac{\cos x}{\sin x} \cdot \cos x$

$= \sin x + \frac{\cos^2 x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$

Section 5.1 (Day 1) – Practice with Identities!

b) $\frac{\sin\vartheta}{1+\cos\vartheta}+\frac{\cos\vartheta}{\sin\vartheta}$

$$= \frac{\sin\theta(\sin\theta)+\cos\theta(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{\sin^2\theta+\cos^2\theta+\cos\theta}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{1+\cos\theta}{(1+\cos\theta)(\sin\theta)} = \csc\theta$$

We can also use them to change rather than simplify:

c) Rewrite $\frac{1}{1+\sin x}$ so it is NOT in fractional form

$$\frac{1}{1+\sin x}\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{1-\sin x}{1-\sin^2 x}$$

$$= \frac{1-\sin}{\cos^2 x} = \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} - \left(\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\right)$$

$$= \sec^2 x - \tan x \sec x$$

Section 5.1 (Day 1) – Practice with Identities!

Factoring

a) $\sec^2\vartheta - 1$

$= (\sec\theta - 1)(\sec\theta + 1)$

b) $\sin x\cos^2 x - \sin x$

$= \sin x(\cos^2 x - 1)$

$= -\sin x(1 - \cos^2 x)$

$= -\sin x(\sin^2 x)$

$= -\sin^3 x$

c) $4\tan^2\beta + \tan\beta - 3$

$4x^2 + x - 3$

$(4x - 3)(1x + 1)$

$= (4\tan\beta - 3)(\tan\beta + 1)$

d) $\csc^2 x - \cot x - 3$

$= 1 + \cot^2 x - \cot x - 3$

$= \cot^2 x - \cot x - 2$

$= (\cot x + 1)(\cot x - 2)$

e) $\tan^2 x - \tan^2 x\sin^2 x$

$= \tan^2 x(1 - \sin^2 x)$

$= \tan^2 x\cos^2 x$

$= \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x = \sin^2 x$

Homework: p. 379 #1, 5, 6, 10, 15-18, 27-32